New Three Dimensional Stability Criteria for Circumbinary Planets

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Abstract: In this work we revisit the problem of the stability of circumbinary planetary orbits. We perform numerical integrations of more than 3 10^8 circumbinary systems over 10^6 planetary orbital periods. We consider, for the first time, non-zero initial planetary eccentricities up to 0.9. Moreover, our investigation covers a wide range of masses for both the binary and the planet and orbital mutual inclinations ranging from 0 to 180 degrees. The results of the numerical integrations provide us with two critical borders: an outer border beyond which all planetary orbits are stable and an inner border closer to the binary below which all planetary orbits are unstable. In between the two borders, a mixture of stable and unstable planetary orbits is observed. We provide empirical expressions in the form of easy to use fits for these two critical borders. Application of our results to real circumbinary systems is also presented.

1. Introduction

The problem of determining stable orbit configurations within the three body problem is one of the classical problems in Celestial Mechanics. Several exoplanets have been discovered in circumbinary configurations, i.e. the planet orbits the center of mass of the stellar binary. An important aspect of ruling out false positives in the quest for exoplanets is the assessment of whether a predicted orbital configuration is dynamically stable. In this work, we revisit the problem of the stability of circumbinary orbits and we remedy any limitations and inconsistencies that arose in previous studies.

2. Methodology

We carry out numerical simulations of a planet around a stellar binary using a symplectic integrator (Mikkola 1997). The parameter space is sampled as follows:

$$M_b \in \{0.5, 0.3, 0.1, 0.05, 0.02, 0.01\},\ \ M_p \in \{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}\},$$

$$I_m \in \{0^o, 18^o, 36^o, 54^o, 72^o, 90^o, 108^o, 126^o, 144^o, 162^o, 180^o\},\ \ e_b, e_p \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\},$$

$$\Omega_p, \omega_p, \varpi_p \in \{0^o, 90^o, 180^o\}, f_b \in \{0^o, 180^o\}, f_p \in \{0^o, 45^o, 90^o, 135^o, 180^o, 225^o, 270^o, 315^o\},\ \ \varpi_b = 0^o$$

where $M_b=m_2/(m_1+m_2)$, $M_p=m_p/(m_1+m_2)$ (m_1 and m_2 are the stellar masses and m_p the mass of the planet). I_m is the mutual inclination, e is the eccentricity, Ω the longitude of the ascending node, ω the argument of pericenter, ϖ the longitude of pericenter, f the true anomaly. The indices b and p refer to the binary and the planet respectively. The integration time was set to 10^6 planetary orbital periods.

A system is classified as unstable if at least one of the following happens: a) any of the eccentricities >1 b) orbit crossing occurs c) $a_b/a_{b0} \leq 1$ d) $a_b/a_{b0} \geq 100$ e) $a_p/a_{b0} \geq 1000$, where a_b and a_p are the two semi-major axes of the system. We define two critical semi-major axes: i) the outer one, above which the planet is stable for all initial positions and ii) the inner one, below which the planet is unstable for all initial positions.

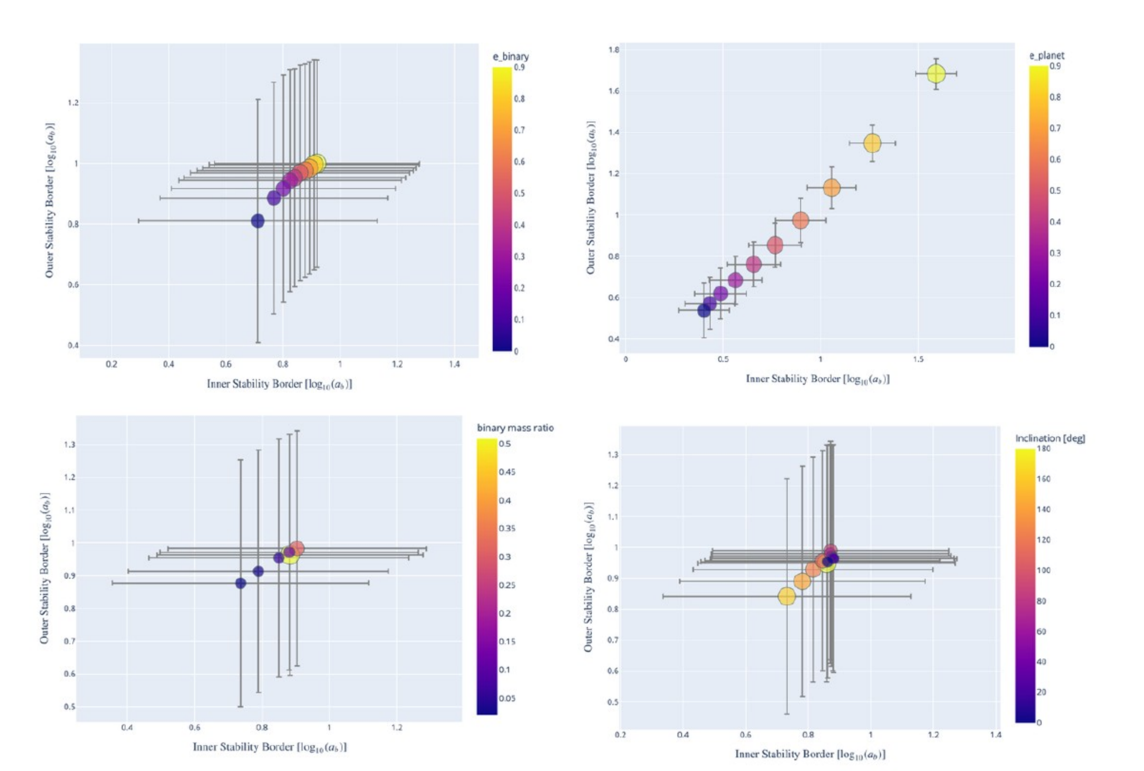
3. Results

<u>Empirical fits</u> (ep ≤ 0.8)

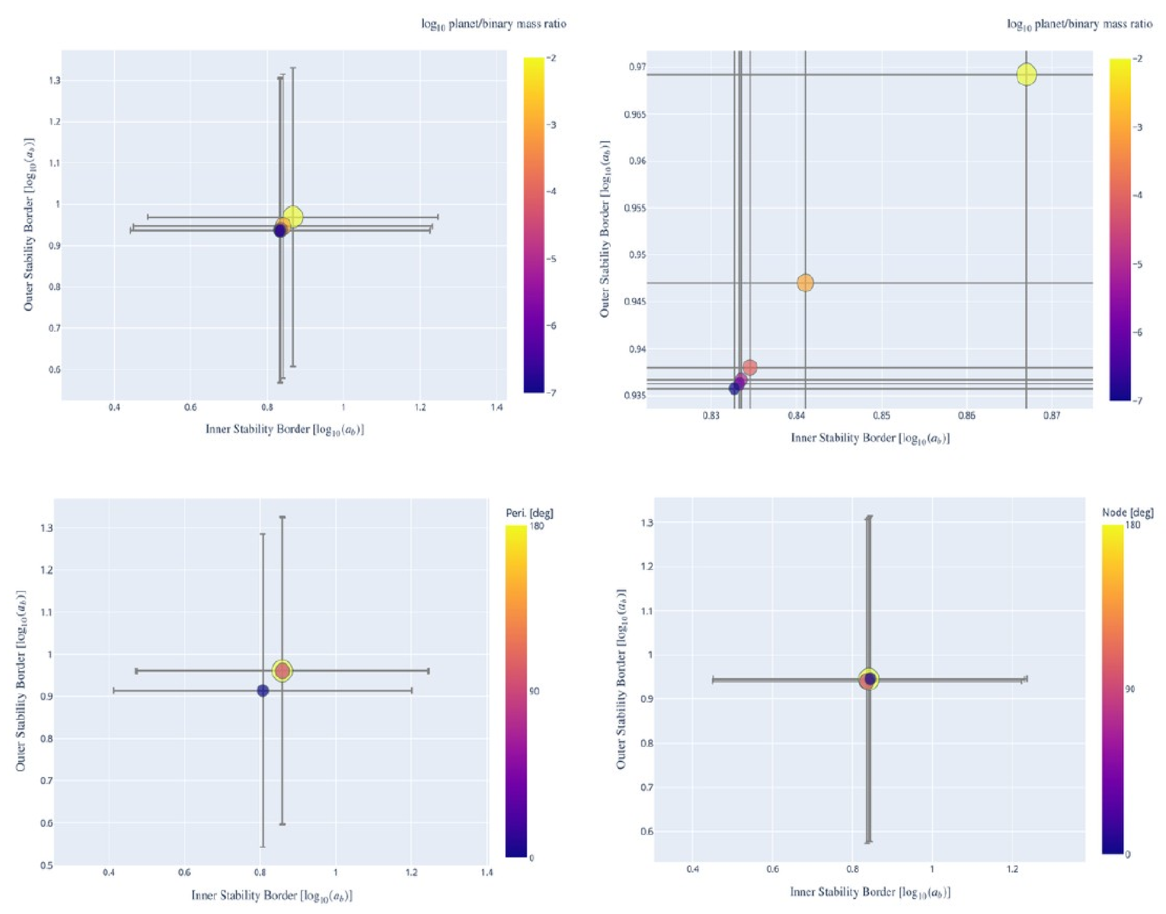
$$a_i^{cr} = \bar{m}\, a_b\, (0.20 - 0.33M_{lb} + 0.10I_m + 0.58e_b + 0.37e_p - 0.26M_{lb}^2 - 0.06I_m^2 + 0.09 - 0.38e_b^2 + 1.02e_p^2 + 0.27M_{lb}e_b + 0.39\, M_{lb}e_p - 0.20\, I_m e_b - 0.25M_{lb}e_b^2 - 0.30M_{lb}e_p^2 + 0.09\, I_m e_b^2 - 0.06M_{lb}^3)$$

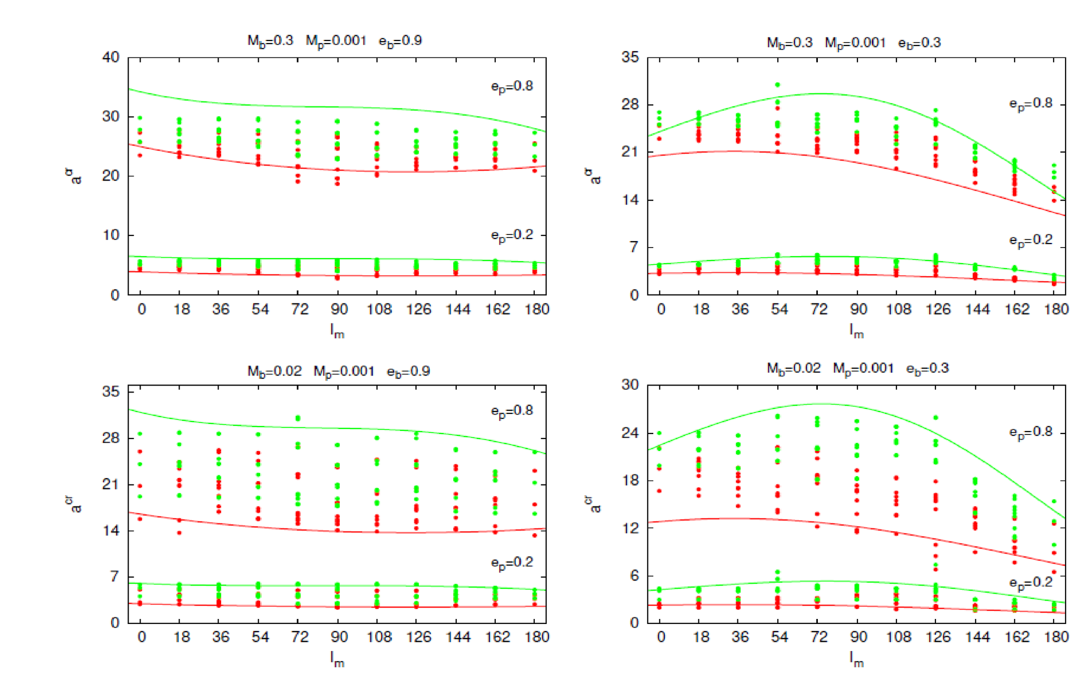
$$a_o^{cr} = \bar{m}\, a_b\, (0.24 - 0.29M_{lb} + 0.23I_m + 1.07e_b + 0.62e_p - 0.22M_{lb}^2 - 0.07I_m^2 + 0.88 - 1.53e_b^2 - 0.47e_p^2 - 0.31I_m e_b - 0.01\, I_m e_p + 0.11\, I_m^2 e_b - 0.04M_{lb}^3 - 0.01I_m^3 + 0.88\, e_b^3 + 1.26e_p^3)$$

$$M_{lb} = log_{10}(M_b) \qquad \bar{m} = [\frac{m_1+m_2+m_p}{m_1+m_2}\frac{1}{1+m_p}]^{1/3}$$



Mean and standard deviation of outer vs inner stability borders in units of $\log_{10}$ of the binary semi-major axis. The color scale refers to the binary orbit eccentricity (top left), the planet's orbital eccentricity (top right), the binary mass ratio (bottom left), and the mutual inclination (bottom right). Stability limits depend strongly on the planetary orbital eccentricity which accounts for most of the variance in the system. Stability borders also show roughly the same sensitivity to the binary star orbital eccentricity, the binary mass ratio as well as the inclination of the system.

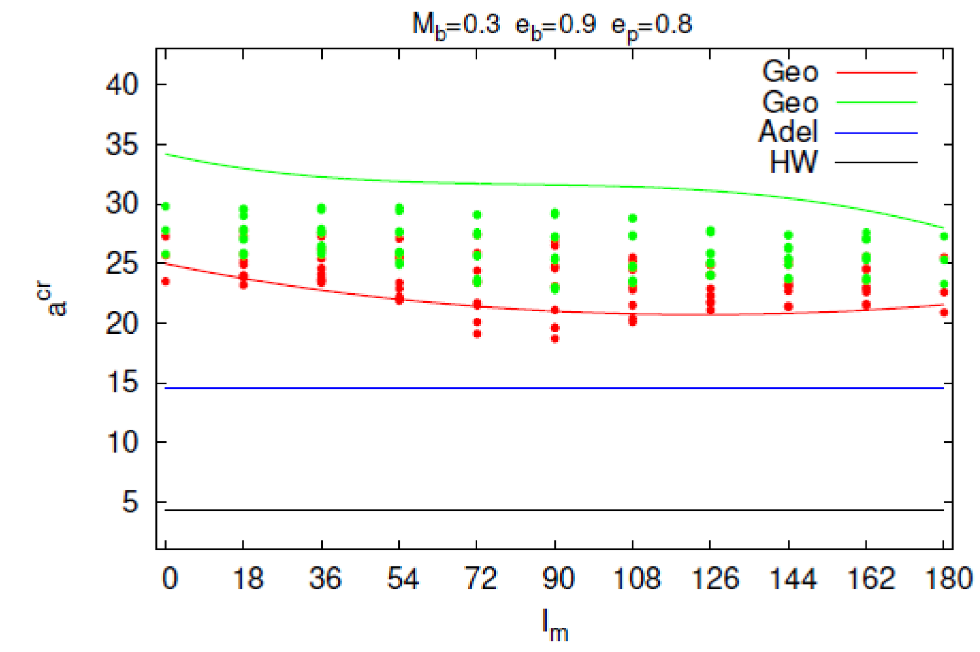




Critical semi-major axis ratio against mutual inclination for a variety of systems. The red color refers to the inner boundary, while the green colour denotes the outer stability border. The continuous lines are our empirical fits

<u>Additionally:</u>

- We provide fits for ep ≤ 0.9
- We train a Machine Learning model
- We compare our empirical formulae against randomly generated circumbinary systems
- We use our fits to characterize known circumbinary systems
- We provide an online tool for stability limits prediction



Geo = this work
Adel = Adelbert et al. (2023)
HW = Holman & Wiegert 1999

References
Mikkola, S., 1997, *CeMDA*, **67**, 145.