Characterizing the transition from stability to instability in compact multi-planet systems

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Abstract:

Understanding stability of exoplanet systems is crucial for constraining planetary evolution and formation theories. We use SPOCK (a machine-learning stability indicator) to characterize the stability of 50 high-multiplicity systems from the California Kepler Survey (CKS). We constrain the range of stable eccentricities for each system. In studying correlations between characteristic eccentricity and various planet-pair and system-level metrics we find that minimum period ratio correlates most strongly with characteristic eccentricity. These results support the idea that chaotic zones near resonance overlaps are an important mechanism in the (in)stability of these systems.

Mapping Stability onto Eccentricity:

We used the machine learning module SPOCK (Tamayo 2020) to investigate the (in)stabilities of observed 3+ planet systems in the CKS sample over a range of eccentricities (e=0-0.12). For every trial, all the planets within a given system had the same eccentricity. The mean anomaly and argument of periastron were drawn from random uniform distributions. **We define the "characteristic eccentricity" to be the eccentricity at which the mean SPOCK stability is 0.5** (ex. Fig. 1). We propagated eccentricities to minimum planet separations in units of mutual hill radii (ex. Fig. 2).

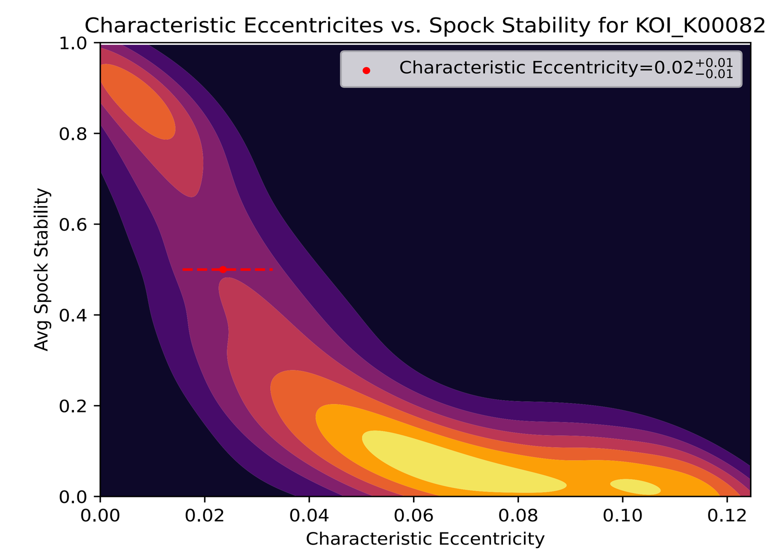

Fig. 1. A KDE plot of tested eccentricities vs. SPOCK stability for the system KOI-82 . At a SPOCK stability of 0.5 maps to a range of eccentricities, of which we take the median as the characteristic eccentricity. This method was applied to all 3+ planet systems in the CKS sample.

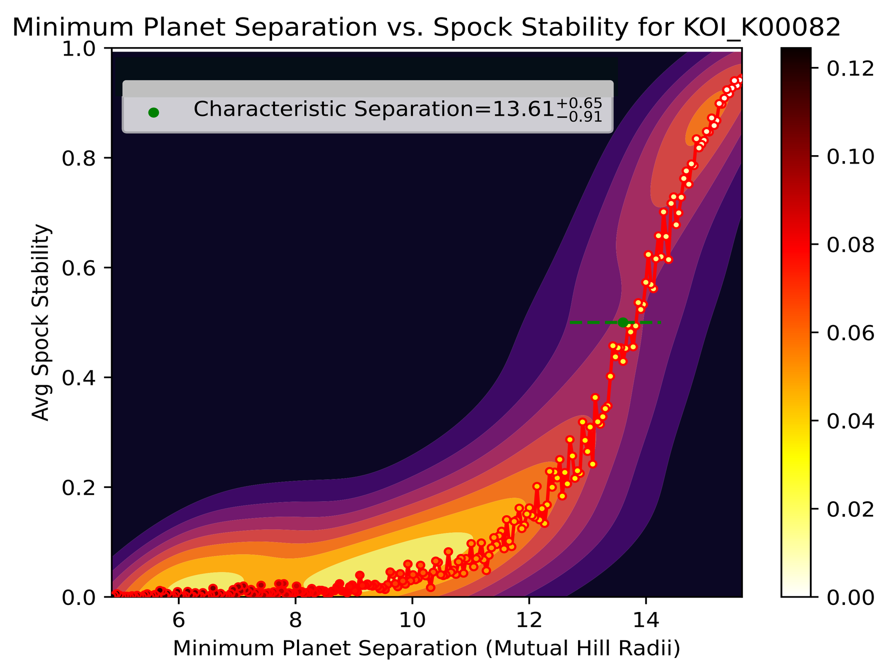

Fig. 2. A KDE plot of minimum separation in units of mutual hill radii vs. SPOCK stability for KOI-82. As expected separation and stability are correlated.

Minimum Period Ratio is the Best Predictor of Characteristic Eccentricity:

The systems we investigate have complex architectures (Fig. 3), and various metrics have been developed to describe them. Many of these metrics correlate with SPOCK stability (Fig. 4). Of these metrics **the strongest correlation is between the minimum period ratio within a system and "characteristic eccentricity"** (Spearman-r=0.79, p-value<10^{-5}). Fig. 3 highlights this correlation by plotting CKS systems ranked by minimum period ratio and colored by eccentricity; note the high-eccentricity systems (yellow) are at the top. Volk and Malhotra 2024 similarly found a relationship between period ratio and stability.

Period Ratios are a strong Predictor for Individual Planet Eccentricities :

We extended our analysis by allowing each planet to have its own eccentricity and **identified characteristic eccentricities for every planet in each system**. We found **planet-pair period ratio correlated with individual planet characteristic eccentricity** (Fig. 5).

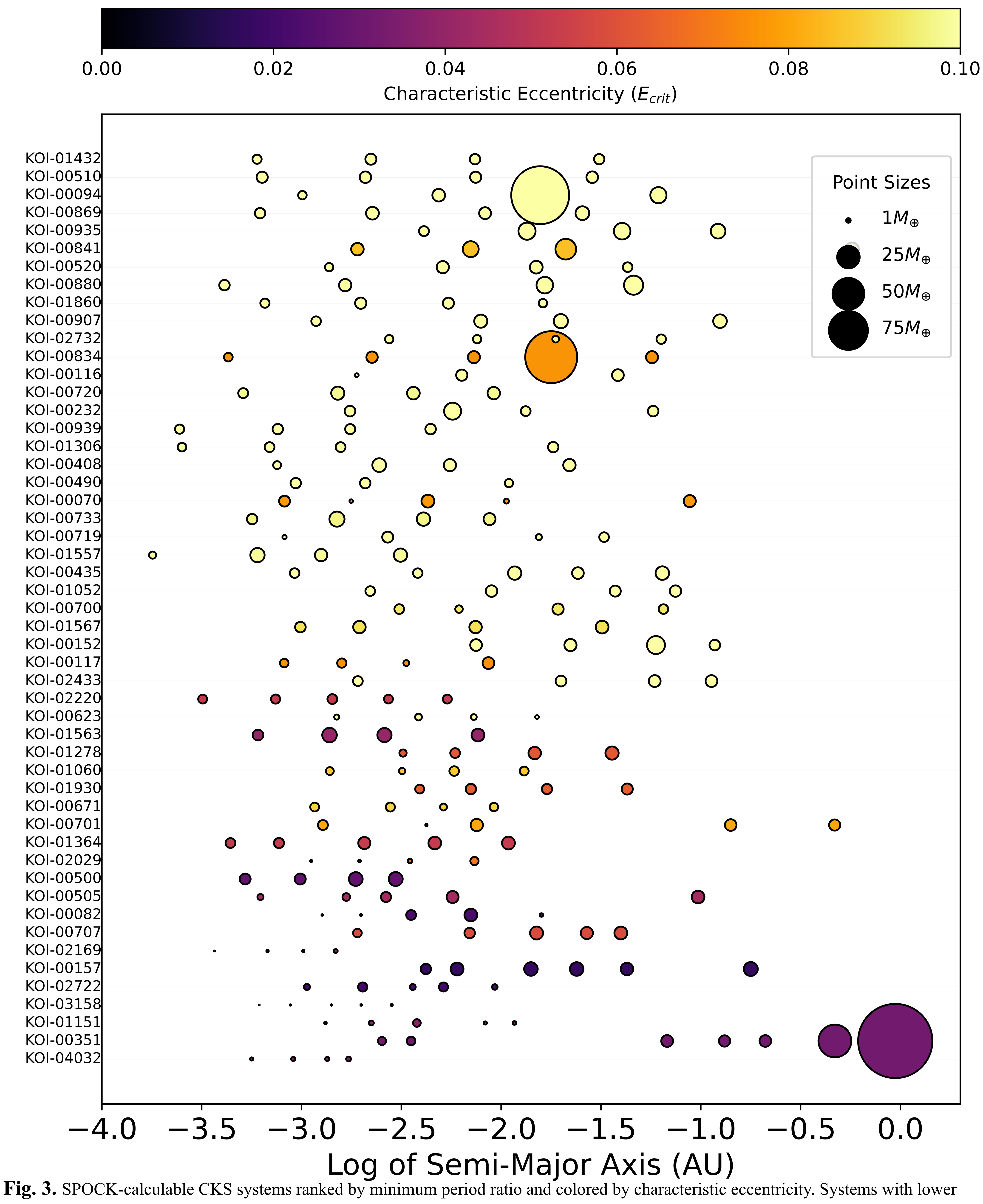

Fig. 3. SPOCK-calculable CKS systems ranked by minimum period ratio and colored by characteristic eccentricity. Systems with lower minimum period ratios (bottom of the plot) have lower eccentricities while systems with higher minimum period ratios (top of the plot) have larger eccentricities.

Metric correlated with eccentricity	p-value	r-value
Minimum Period Ratio	<10^{-5}	0.79
Minimum separation in units of mutual hill radii for the system	<10^{-4}	0.67
Gap Complexity	0.0397	-0.29
Mass Partitioning	0.2034	-0.18
Complexity of spacing in units of mutual hill radii	0.0244	-0.32

Fig. 4. A Table of Spearman r-values and Spearman p-values for minimum period ratio as well as other system metrics.. Spacing complexity in terms of mutual hill radii was also tried, however it did not perform significantly better than other known metrics of similar type.

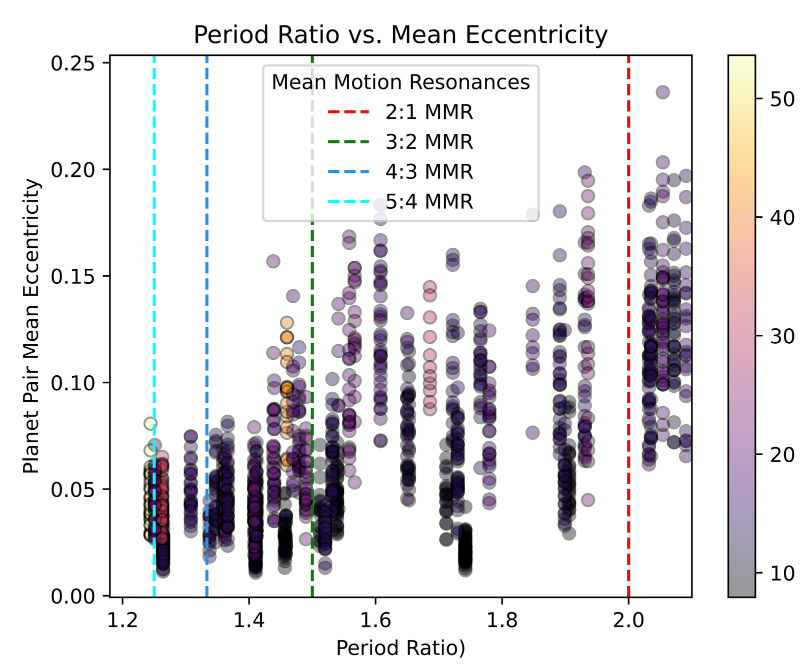

Fig. 5. Period Ratio for a pair of planets vs. the mean of the two planet's characteristic eccentricities. Period ratio correlated with stability. Currently, these results are being compared to necessary eccentricities for MMR overlap

References:
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