

## Introduction

Keplerian-Stacker represents the pioneering multi-epochs algorithm (Le Coroller et al. 2015; Nowak, M. et al. 2018; Le Coroller, H. et al. 2022) designed to combine numerous observations while accommodating the orbital motion of potential exoplanets within the images to enhance the final detection threshold. These last years, K-Stacker has seen substantial enhancements, notably a thousandfold increase in computational speed. This acceleration has been achieved by employing the efficient Python Kepler module. Additionally, several computational components have been optimized through a reimplemention in Cython. The likelihood function has also been rewritten considering a Gaussian noise :

$$\text{Log}(L) = 0.5 \frac{\left( \sum_{i=0}^n \frac{S_i}{\sigma_i^2} \right)^2}{\sum_{i=0}^n \left( \frac{1}{\sigma_i} \right)^2} - 0.5 \sum_{i=0}^n (\text{SNR}_i)^2 \quad \text{Where } S_i \text{ is the signal and } \sigma_i^2 \text{ the variance at the position given by the Kepler equation in image i.e. } \text{SNR}_i = \frac{S_i}{\sigma_i}$$

The first term of this equation is the log of the probability to have a planet in the data, while the second term is the log of the probability to don't have any planets in the data.

## Contrast limit reached by K-Stacker on SPHERE ZIMPOL observations of eps Eridani

K-Stacker generates a probabilistic detection map. Figure 1, illustrates that the detection of epsi Eri b, with an approximate radius of 0.8 Jupiter radii, was not achievable in the 12 SPHERE-Zimpol observations (Tschudi, C. et al. 2024, A&A). However, K-Stacker has enhanced the contrast limit by a factor equal to the square root of the number of observations (x3.5).

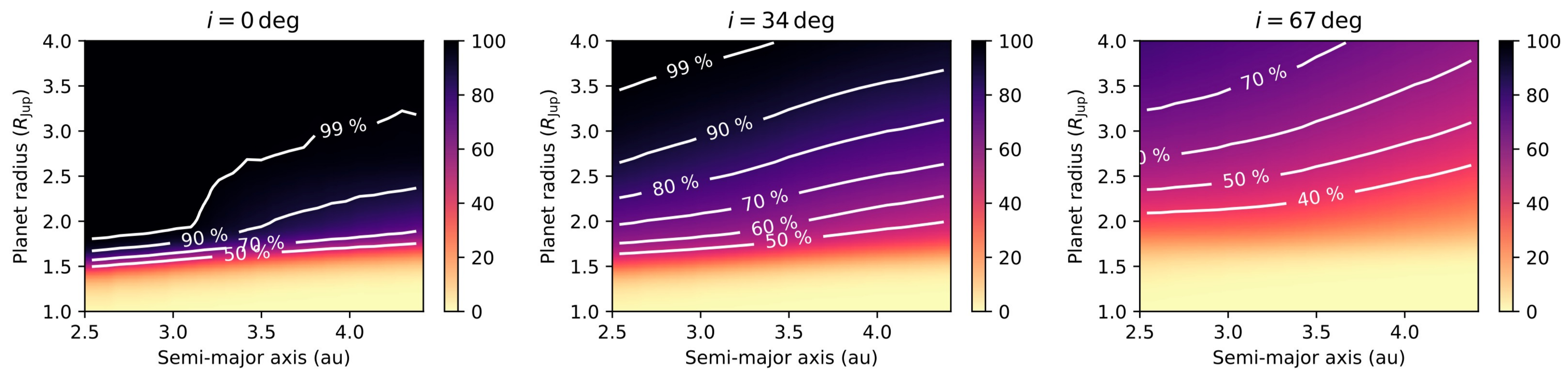


Fig. 1 Detectability of planets around epsi Eridani for which a planet of given radius would have been detected by K-Stacker, for 3 different values of the inclination. The inclination has a significant impact on the detectability of planets, due to the influence of the phase angle on the polarization contrast.

## Estimation of the K-Stacker detection Limit on simulated Harmoni observations

We conducted simulations of observations in the HK band using the SP2 apodizer, which has been extensively examined in the thesis of Houllé, M. for benchmarking purposes. Our simulations began under optimal seeing conditions (JQ1 configuration) to ascertain the maximum achievable contrasts. The harmoni\_hc\_photometry.py module (Houllé, M. 2021, A&A, 652) has been adapted to inject synthetic planets at varying astrometric positions across four epochs. These planets were integrated into simulated observations of HD 95086 to emulate the search for a companion, 'c' (Desgrange, C. et al. 2022, A&A, 664). Each observation has been reduced by a PCA-ASDI algorithm with the VIP package (Gonzalez, G. et al. 2017, Christiaens et al. 2023).

### Injection of planets:

Orbit / # Planet	a (au)	e	t0 (Yr)	Ω (deg)	i (deg)	w (deg)
#1	12	0	18	0	0	0
#2	16	0.5	22	57	45	37
#3	18	0	40	0	0	0
#4	23	0	10	0	0	0

Table 1: Orbital parameters of the planets injected in the simulated Harmoni data

### Detections with K-Stacker:

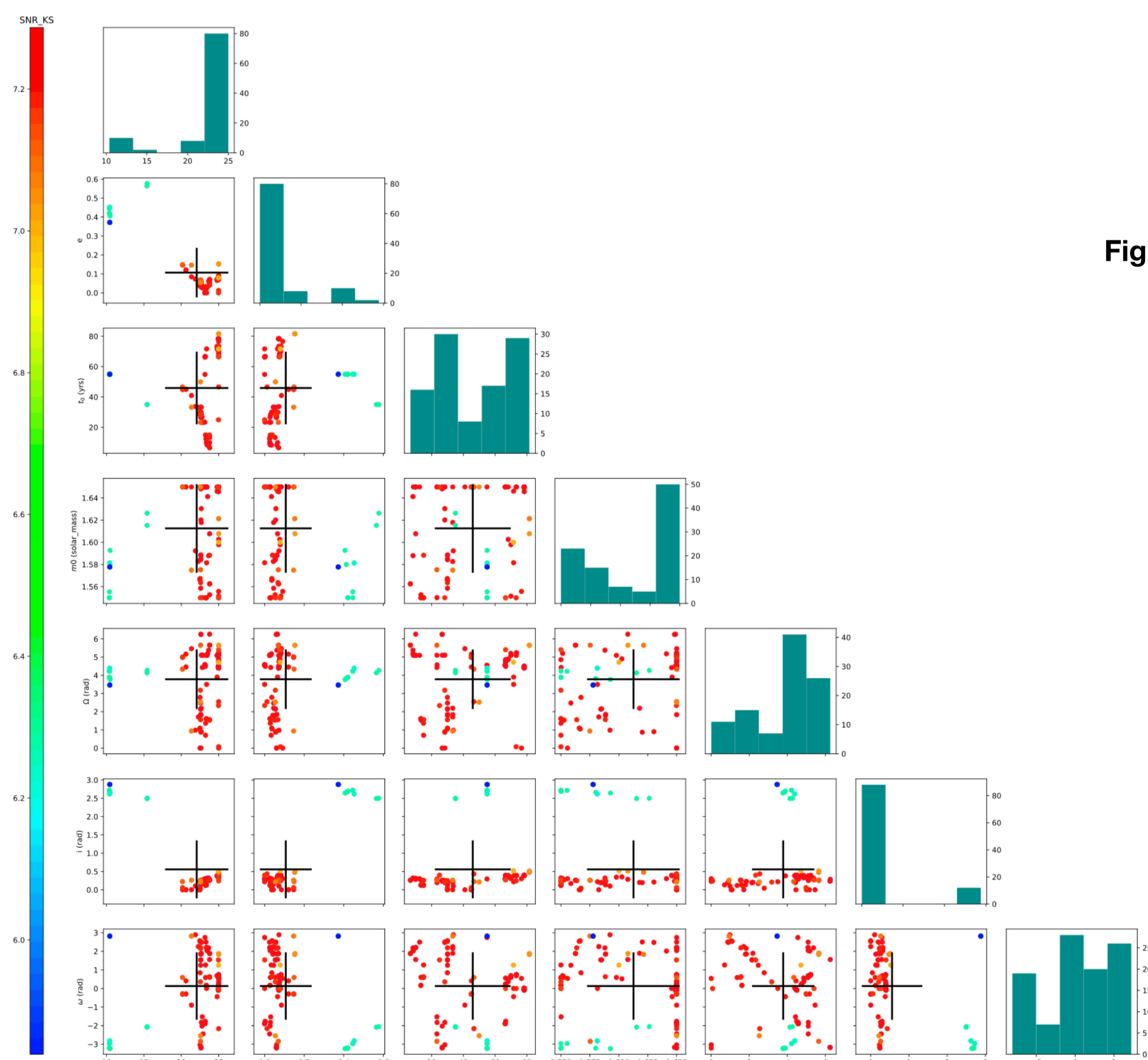


Fig. 4: Illustrative corner plot of the orbital parameters for planet #4 at dmag=17, as determined by K-Stacker. The parameters are accurately retrieved, with the exception of the time of periastron passage (t0) and argument of periastron (w), which are typically redundant in the context of a face-on orbit.

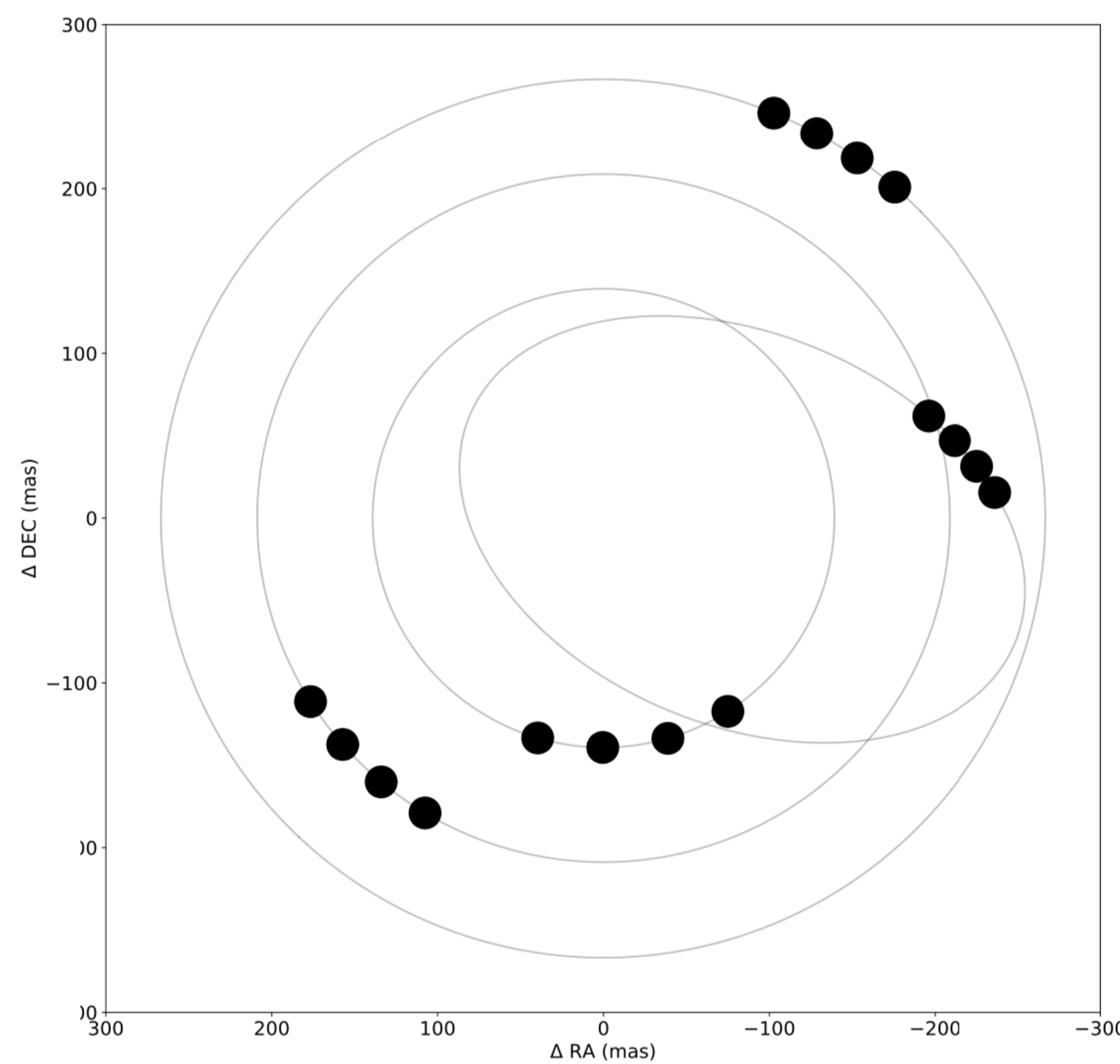


Fig. 2: plot of the orbits corresponding to parameters of table 1

orbit / contrast (dmag)	#1	#2	#3	#4
17	No	Yes	Yes	Yes
16	No	Yes	Yes	yes
15	Yes	Yes	Yes	yes

Table 2: Planets detected by K-Stacker

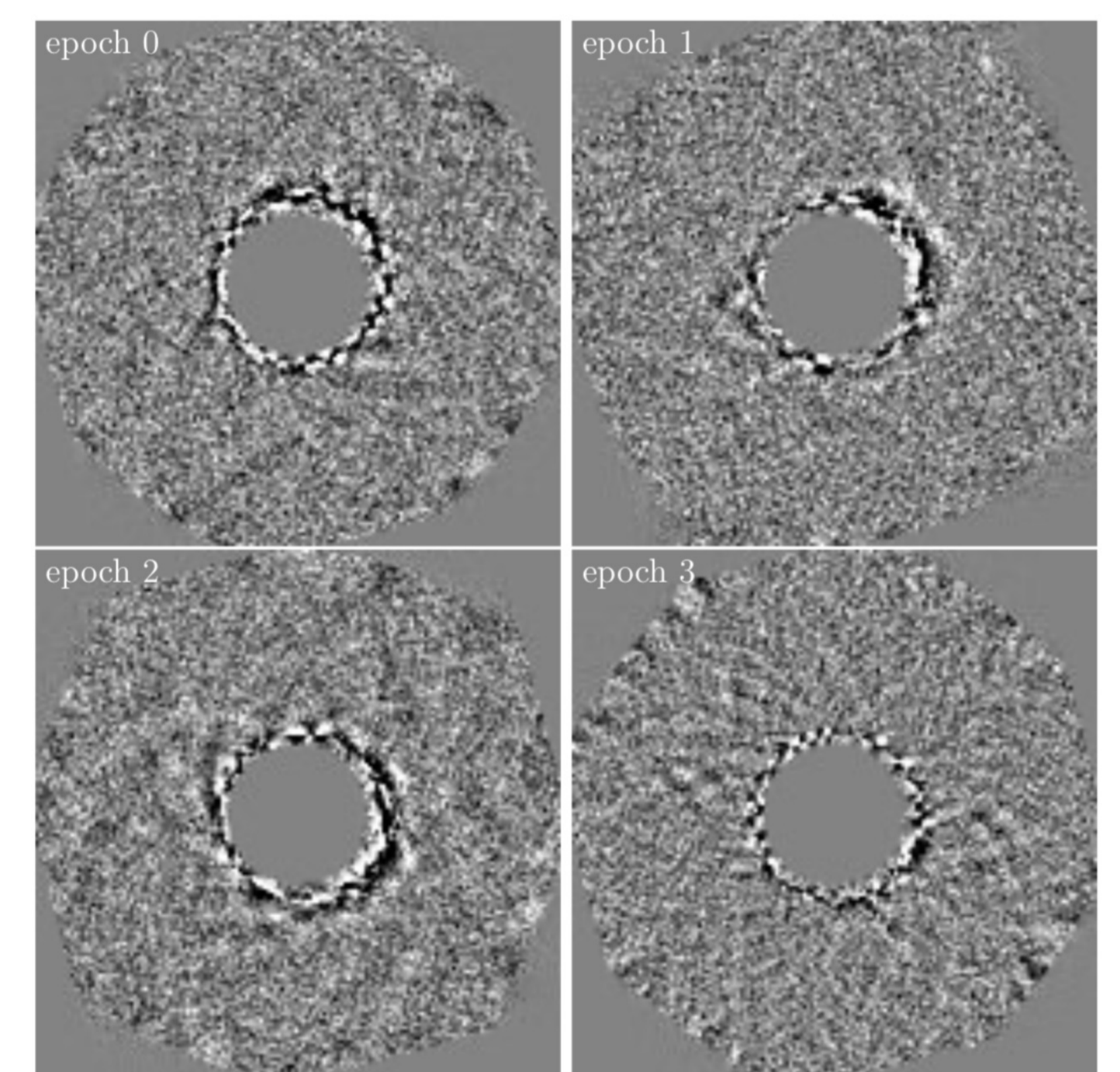


Fig. 3: Reduced PCA-ASDI observations with VIP, wherein planet #2, with dmag=17, remains undetected in individual epochs at snr ≈ 3.

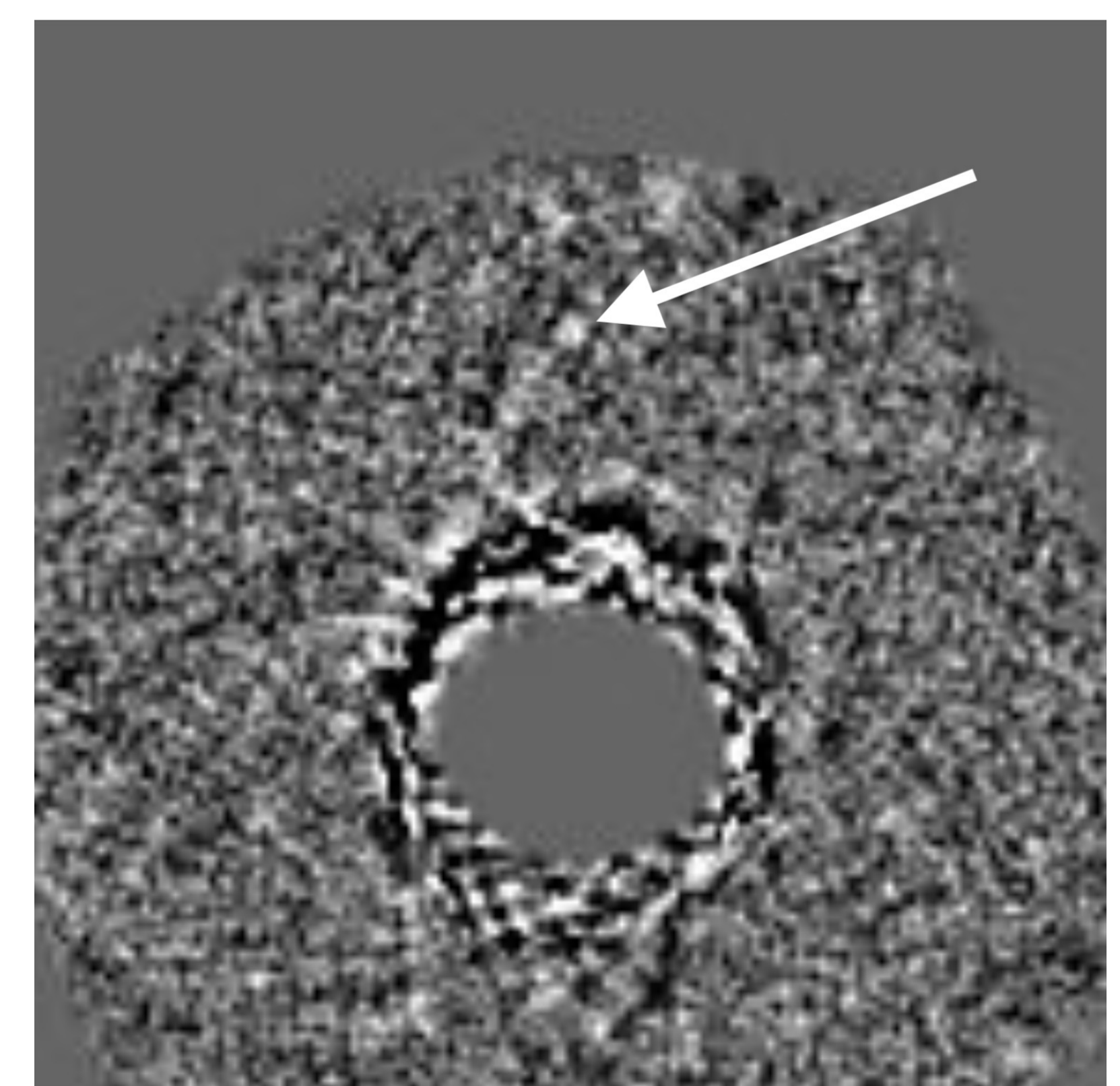


Fig. 5: Planet #2, with a differential magnitude (dmag) of 17, is successfully detected at snr\_ks = 6.2 by K-Stacker using the Reduced images displayed in Figure 3.

## Conclusion

K-Stacker approach significantly enhances the detection limit of direct imaging techniques. We highlight a practical application of K-Stacker, allowing to probe planets smaller than 2 Jupiter radius in the eps Eri area at less than 4 a.u., using the SPHERE ZIMPOL instrument. Additionally, we showcase the potential of K-Stacker in the context of future astronomical instruments like HARMONI on the E-ELT. Through simulated images, we demonstrate how K-Stacker can exploit the full capabilities of these instrument, pushing the boundaries of our exoplanet detection limits and revealing the orbital parameters of the detected planets. K-Stacker is more than just an algorithm: it represents a novel observational strategy in high-contrast imaging that minimizes telescope time while achieving the highest contrasts and extracting unbiased orbital parameters. K-Stacker is on git (<https://github.com/kstacker/>) and can be easily installed such as a python package [pip install]. It can run on any high contrast reduced images (PCA-ASDI, TLOCI, etc.). We regularly improve our code to increase the computational speed, and the statical analysis of the results.

## Publications

Christiaens, V., et al. 2023, JOSS, "VIP: A Python package for high-contrast imaging"  
Gonzalez, G. et al. 2017, ApJ, "VIP: Vortex Image Processing Package for High-contrast Direct Imaging", 154  
Houllé et al. 2021, A&A, "Direct imaging and spectroscopy of exoplanets with the ELT/HARMONI high-contrast module" 652, A67  
Le Coroller et al. 2015, American Astronomical Society OHP2015, K-Stacker, a new way of detecting and characterizing exoplanets with high contrast imaging instruments, V. 47  
Nowak, M. et al. 2018, A&A, "K-Stacker: Keplerian image recombination for the direct detection of exoplanets", 615, 144

K-Stacker can be applied on any high contrast imaging instrument data:

VISIR: Le Coroller, H. et al. 2022, A&A, "Efficiently combining Alpha CenA multi-epoch high-contrast imaging data", 667, 142  
SPHERE-IRDIS/IFS: Le Coroller, H. et al. 2020, A&A, "K-Stacker: an algorithm to hack the orbital parameters of planets hidden in high-contrast imaging", 639, 113  
SPHERE-Zimpol: Tschudi, C., Schmid, H.M., Nowak, M., et al. 2024, "SPHERE RefPlanets: Search for eps Eridani b and warm dust", accepted

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Contact us, if you would like we run K-Stacker on your multi-epochs data:  
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## K-Stacker Github

