Global 3D MHD simulations of Jupiter-like dynamos at different evolutionary times

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Introduction

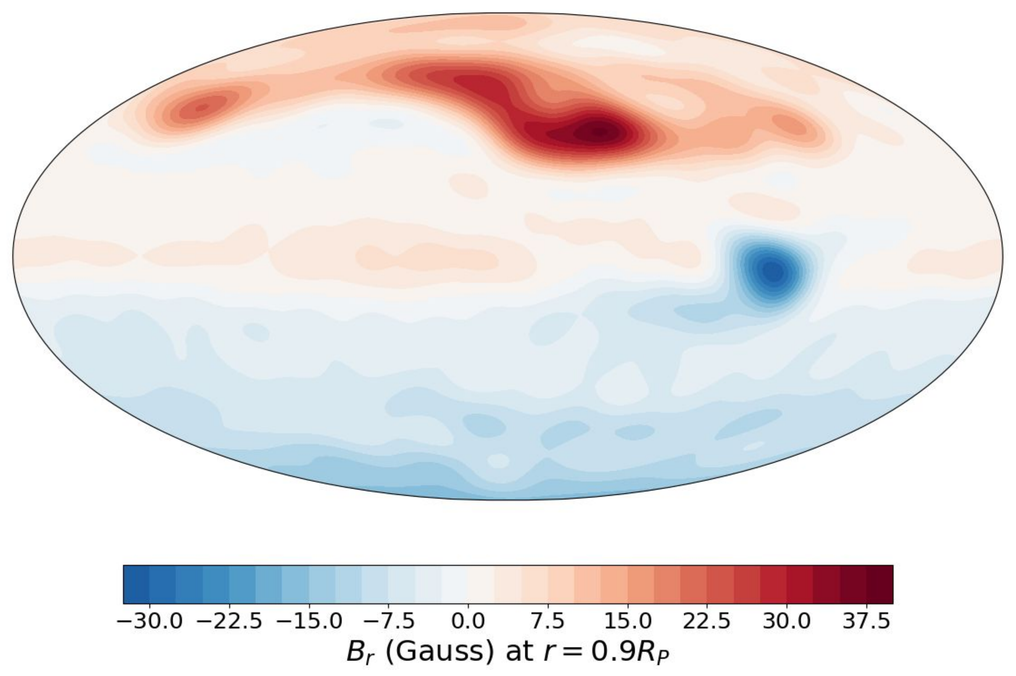


Figure: Radial component the magnetic field of Jupiter reconstructed using data from Connernay et al. (2021).

More than 5500 exoplanets have been discovered so far, but their magnetism is still unknown. Based on knowledge of the Solar System planets, we expect giants planets to host the strongest magnetic fields and thus the easiest to detect. Here we want to address the long-term (Gyr) evolutionary changes of the convection driven interior dynamos of Jupiter-like planets by using MHD simulations. **The goal is the understanding of intensity and topology evolution of magnetic fields in gas giants interiors.**

Method

Time evolve (in a Gy timescale) a **1D hydrostatic model** of a gas giant with **MESA** → Implement the radial profiles as the **background state** of an **anelastic** 3D model → Time evolve (in a Ky timescale) the **3D MHD equations** with MagIC in spherical shell to obtain **self sustained dynamo solution** → Repeat the process for **multiple stages of the life** of a gas giant

1D MESA hydrostatic profiles

To model the change of radially dependent thermodynamic quantities of gas giants we use the publicly available 1D code MESA[a]. It solves the time-dependant stellar structure equations: the **mass** and **energy conservation**, **hydrostatic equilibrium**, **energy transport**, and EoS of H-He mixtures in the gas-giant regime (Paxton et al., 2019, Saumon et al., 1995).

$$\frac{dm}{dr} = 4\pi r^2 \rho\,, \tag{6}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}\,, \tag{7}$$

$$\frac{dL}{dm} = \epsilon_{\rm grav} + \epsilon_{\rm irr} + \epsilon_{\rm dep}\,, \tag{8}$$

$$\frac{dT}{dm} = -\frac{GmT}{4\pi r^4 P}\nabla \tag{9}$$

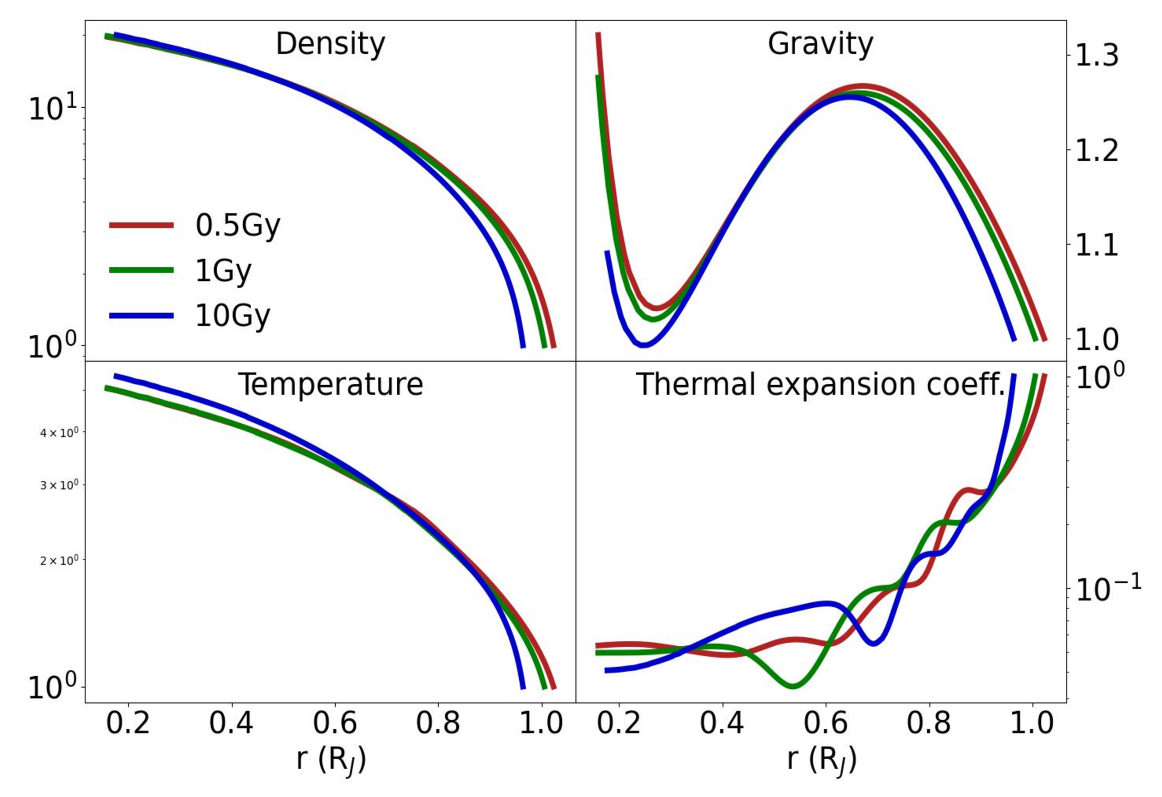


Figure: Normalized MESA hydrostatic quantities at three different evolutionary times for a $1M_J$ planet. The outer parts of the profiles have been cut such that ϱ_o/ϱ_i is at most 20. They are implemented in MagIC as **high degree polynomial fits.** The real physical units are used for the dimensionless parameters and code unit assignment. The thermal expansion coefficient and the Grüneisen parameter (not shown) are used in the EoS.

3D Magnetohydrodynamic (MHD) model

We run **3D MHD numerical simulations** using the public code **MagIC**[b], spherical harmonics in θ,φ and Chebyshev polynomials in r) under the **anelastic approximation,** typically used for modeling convection in gas giants and stars. We time-evolve the **mass continuity**, **momentum**, the **induction** and **entropy** equations, respectively:

$$\nabla\cdot(\tilde\rho\mathbf{u}) = 0, \tag{1}$$

$$\frac{\partial\mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{1}{E}\nabla\left(\frac{p'}{\tilde\rho}\right) - \frac{2}{E}\mathbf{e_z}\times\mathbf{u} + \frac{Ra}{Pr}\tilde g\, s\,\mathbf{e_r} + \frac{1}{Pm_i\,E\,\tilde\rho}(\nabla\times\mathbf{B})\times\mathbf{B} + \frac{1}{\tilde\rho}\nabla\cdot\mathbf{S}, \tag{2}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) - \frac{1}{Pm}\nabla\times\left(\tilde\lambda_{norm}(r)\nabla\times\mathbf{B}\right), \qquad \nabla\cdot\mathbf{B} = 0, \tag{3}$$

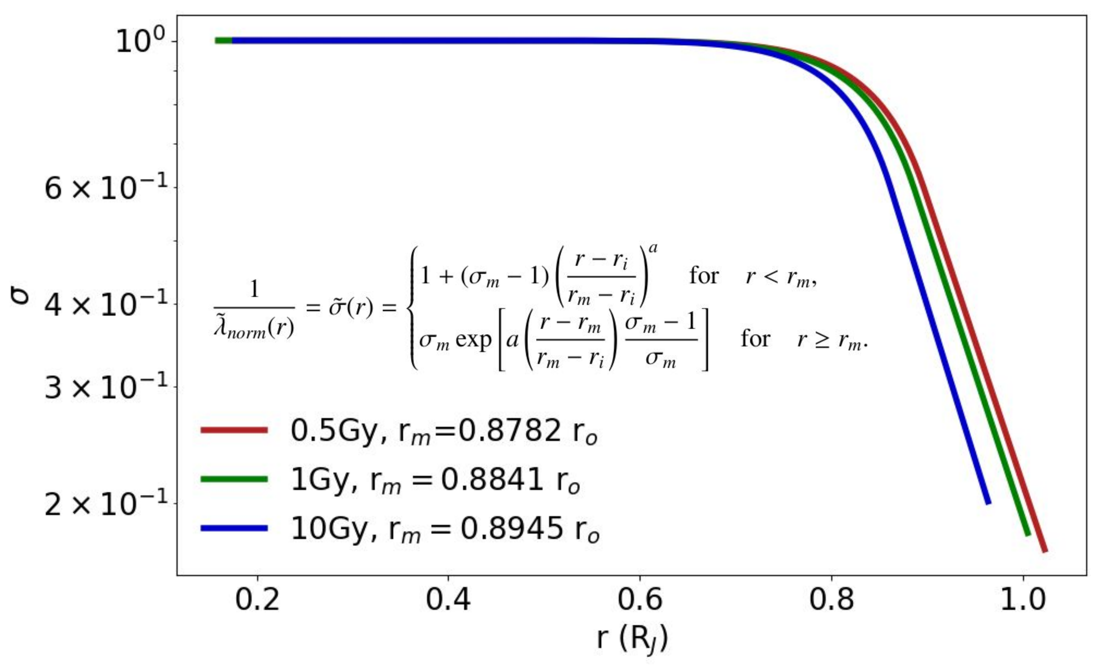
$$\tilde\rho\tilde T\left(\frac{\partial s}{\partial t} + \mathbf{u}\cdot\nabla s\right) = \frac{1}{Pr}\nabla\cdot\left(\tilde\rho\tilde T\nabla s\right) + \frac{Pr\,Di}{Ra}Q_\nu + \frac{Pr\,Di}{Pm_i^2\,E\,Ra}\tilde\lambda_{norm}(\nabla\times\mathbf{B})^2, \tag{4}$$

where

$$E = \frac{\nu}{\Omega d^2} \qquad Ra = \frac{g_o d^3 \Delta s}{c_p \nu\kappa} \qquad Pr = \frac{\nu}{\kappa} \qquad Pm_i = \frac{\nu}{\lambda_i}$$

The dimensionless numbers are the **Ekman**, the **Rayleigh**, the **Prandtl** and **magnetic Prandtl** numbers. The quantities with a tilde are the radially dependent hydrostatic state (taken from MESA except for the magnetic diffusivity $\lambda_{\rm norm}$) used as the static anelastic background.

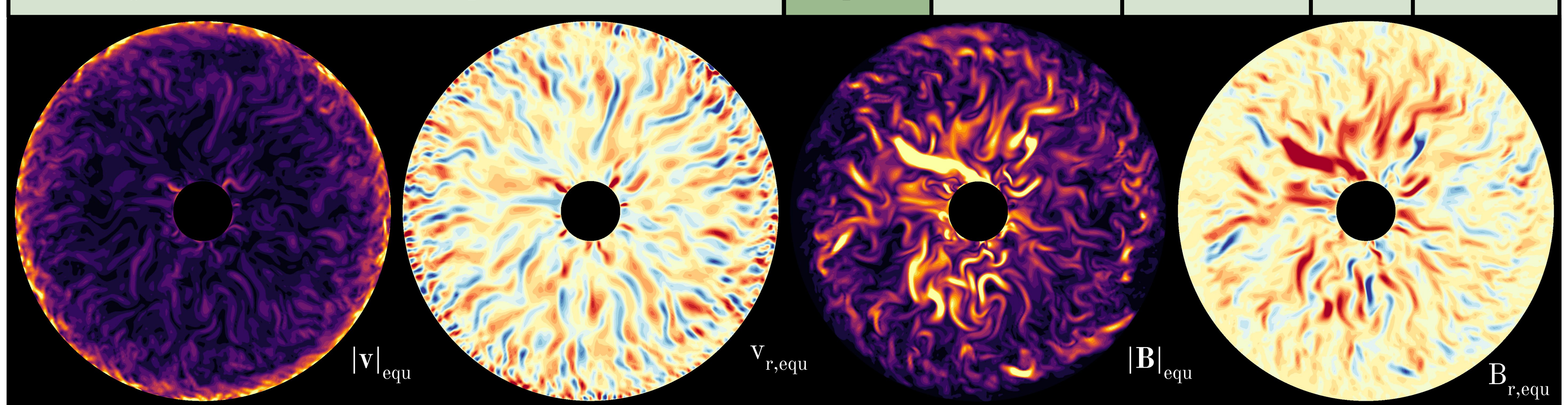
Figure: Normalized the magnetic conductivity σ, i.e. inverse of $\lambda_{\rm norm}$, obtained from Gómez-Pérez et al. (2010) which is similar to jovian interior structure models (French et al., 2012). For the models shown here we use $\sigma_m = 0.6$, $a = 11$ and r_m the radius where MESA pressure reaches just above 100 GPa (**metallic hydrogen transition**). For every 1D model r_m changes, but usually falls between 0.85 and 0.90 outer r



Dynamo parameters and general behaviour

In general, for a given **E, Pr** and **Pm**, one can increase **Ra** until criticality and convection and dynamo develop. We use different dimensionless numbers that reflect the evolutionary changes dictated by MESA to obtain dynamo solutions (despite usual caveat of being orders of magnitude away from the physical values).

Model	E	Ra	Pm	Pr
0.5 Gy	$1.08\cdot10^{-5}$	$1.22\cdot10^{9}$	1	1
1 Gy	$1.12\cdot10^{-5}$	$9.81\cdot10^{8}$	1	1
10 Gy	$1.30\cdot10^{-5}$	$4.97\cdot10^{8}$	1	1
Jupiter	10^{-18}	10^{31}	10^{-6}	10^{-2} - 1



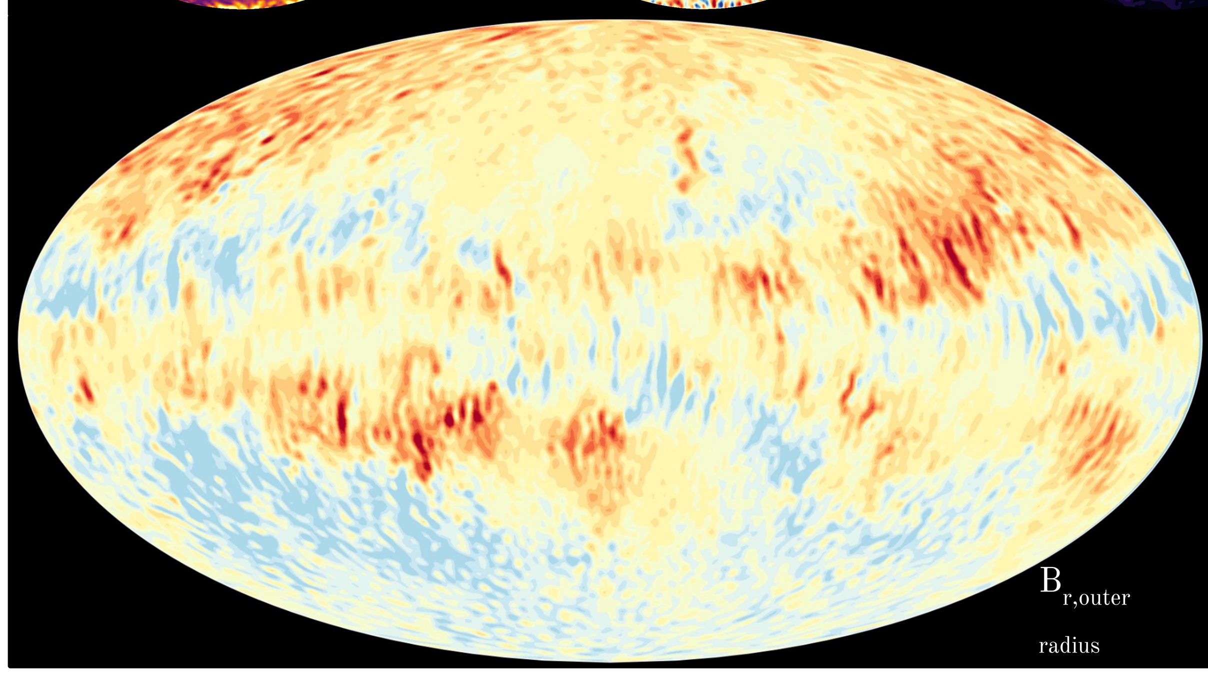


Figure: Equatorial (top) and outer surface (left) snapshots of the 0.5 Gy model. All three cases have Ra high enough to **develop convection all over the volume.** This leads to amplification of **B**, which, during saturation, always develops **similar topology** although reaching **different magnitudes** (see radial distribution and energy spectra).

Diagnostics, radial distribution and spectra

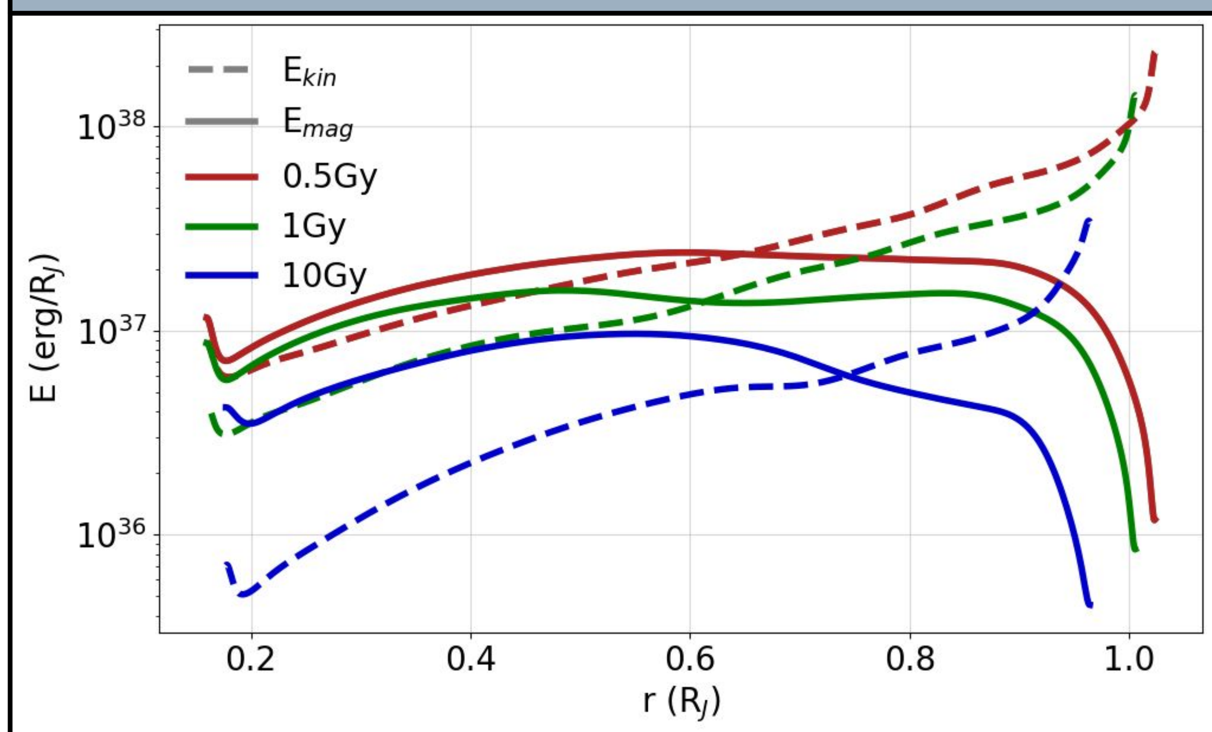


Figure: Radial distribution $E_{\rm kin}$ and $E_{\rm mag}$. The external spikes (i.e. high $E_{\rm kin}$ with reduced $E_{\rm mag}$) are due to the appearance of an outer equatorial latitudinal jets. They are a common trait with models having both stress free boundary conditions and a decaying outer σ. There is a general trend to decay in time, which needs to be compared with numerical scaling laws (Yadav et al., 2013).

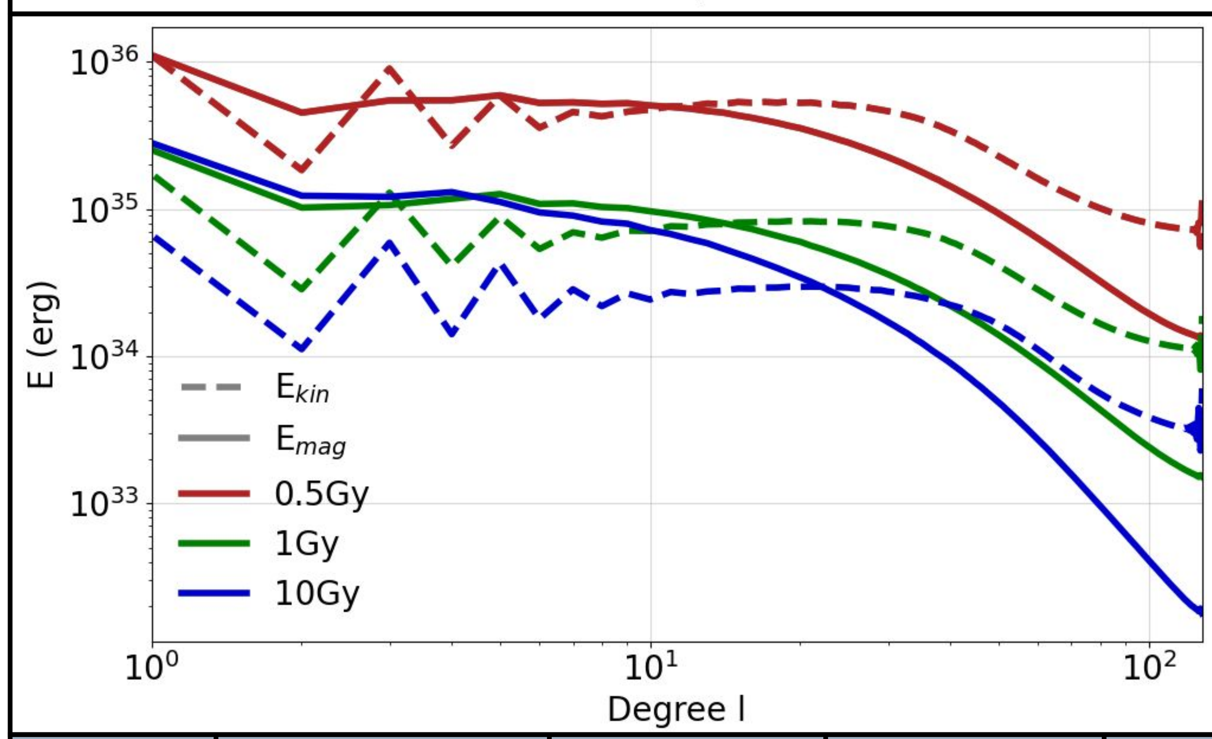


Figure: Magnetic and kinetic energy spectra as function of harmonic degree. The kinetic spectra is a bit under resolved, as the inertial range does not fall more than one order or magnitude. Both the dipole and the magnetic energy seem to increase over time (see f_{dip} and $E_{\rm mag}/E_{\rm kin}$). Generally, planetary parameters do not vary more than one order of magnitude.

Model	Re, Rm	Ro	Λ	$E_{\rm mag}/E_{\rm kin}$	f_{ohm}	f_{dip}
0.5 Gy	$1.020\cdot10^{3}$	$1.002\cdot10^{-1}$	2.59	0.60	0.38	0.37
1 Gy	$7.88\cdot10^{2}$	$8.28\cdot10^{-2}$	1.77	0.65	0.35	0.35
10 Gy	$3.76\cdot10^{2}$	$4.87\cdot10^{-2}$	1.11	1.29	0.34	0.46
Jupiter	$O(10^{12})$, $O(10^{6})$	$O(10^{-6})$	$O(10^{1}$ - $10^{2})$	$O(10^{2}$ - $10^{3})$	~1	0.75

Conclusions

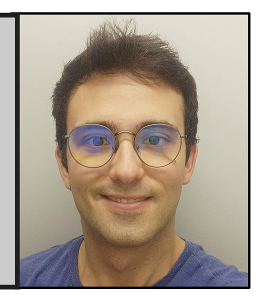
1. Few (3 or 4) stages are enough to asses the general behaviour.
2. In the tested models, the evolution of physical parameters leads to **saturated dynamo solutions** throughout a planets life time.
3. As the planet evolves and cools down **Rm**, **Ro** and **Λ** decrease, but **equipartition level and dipolarity seem to increase.**
4. Power generation (buoyancy) vs dissipation (viscous and Ohmic) remains approximately constant, as seen with f_{ohm}.

Further work

1. Explore distinct **Pm**, **Pr** and **density ratios.**
2. **Increase resolution** so to find a wider inertial range in the kinetic spectra.
3. Apply the same method for the **Hot Jupiter case** by using heated interior models.

References

[1] Connernay et al. JGR Planets 127, 2, 2021
[2] French et al., ApJS 202, 5, 2012
[3] Gómez-Pérez et al., PEPI 181, 1. 42–53, 2010
[4] Paxton et al., ApJS 243, 10, 2019
[5] Saumon et al., ApJS 99, 713-741, 1995
[6] Yadav et al., ApJS 774, 6, 2013
[a] https://github.com/MESAHub/mesa
[b] https://github.com/magic-sph/magic



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